Guide to Finding Asymptotes and Graphing Rational Functions

Original rational function: $f(x) =$ _____

Factor and reduce f.

Simplified rational function: $f(x) =$ _____

* Format Caution: be sure points are written as ordered pairs and lines are given using their appropriate linear equation (in slope intercept form where possible).

The **domain** of f is all $\Re$eal $x \neq$ _____.
(Set denominator of original f to zero and solve for x.)

The graph of f has **hole(s)** at _____.
(Set factor(s) that cancelled out in the simplification process to zero and solve for x. Plug that x into simplified f to get y-coordinate.)

The graph of f has **y-intercept point** at _____.
(Use original (or simplified) f. Let x=0 and solve for y.)

The graph of f has **x-intercept point(s)** at _____.
(Use simplified f. Let y=0 and solve for x. Note that a fraction is equal to zero IFF its numerator is, so you may set the numerator of simplified f to zero and solve for x.)

The graph of f has **vertical asymptote line(s)** at _____.
(Set denominator of simplified f to zero and solve for x.)

The graph of f has **horizontal asymptote line(s)** at _____.
(Use the original f. Let n=degree of numerator, d=degree of denominator, a=coefficient of the highest powered term in the numerator, b=coefficient of the highest powered term in the denominator. Now,
n<d → x-axis (line y=0) is the horizontal asymptote line.
n=d → line y=a/b is the horizontal asymptote line.
n>d → no horizontal asymptote line, but possibly a slant asymptote, see below.)

The graph of f has **oblique asymptote line(s)** at _____.
(Use the original f. If the degree of the numerator is exactly one more than the degree of the denominator (i.e., if n=d+1), then there is a slant asymptote line. To find it perform the long division as indicated by the original f. Ignoring the remainder, the oblique asymptote is the line "y=answer from the division" in slope intercept form.)

Carefully **graph** $f(x)$.

Plot and label axis intercepts, holes, and asymptote lines. If necessary, use a sign chart to determine where f is above or below the x-axis.