

## Trigonometric Identities & Formulas

### Pythagorean Identities:

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

### Sum & Difference Identities:

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

### Double-Angle Identities:

$$\sin 2\theta = 2 \cos \theta \sin \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 1 - 2 \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Laws of Sines & Cosines:

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

### Half-Angle Identities:

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2} \Rightarrow \sin \frac{\beta}{2} = \pm \sqrt{\frac{1 - \cos \beta}{2}}$$

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2} \Rightarrow \cos \frac{\beta}{2} = \pm \sqrt{\frac{1 + \cos \beta}{2}}$$

$$\tan^2 \alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} \Rightarrow \tan \frac{\beta}{2} = \pm \sqrt{\frac{1 - \cos \beta}{1 + \cos \beta}} = \frac{1 - \cos \beta}{\sin \beta} = \frac{\sin \beta}{1 + \cos \beta}$$

### Product Formulas:

$$2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

$$2 \cos \alpha \cos \beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

$$2 \sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$$

$$2 \cos \alpha \sin \beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$$

### Factoring Formulas:

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$