

# If you are planning to take Math 221 - Differential Equations - next term:

Based on surveys of previous Math 221 students, the Mathematics Division and your fellow students want you to know that you should understand and have mastered all of the following topics and sample review questions in order to succeed. However, this is only a sample of the important topics that are expected prerequisite knowledge for Math 221. The ability to complete this small sample of problems does not indicate complete readiness for the class. These questions should be used as a guideline for the basic types of problems to review before the first day of classes.

## Algebra topics to review:

- Partial fraction decomposition
- Completion of the square
- Properties of logarithms
- Polynomial equations

## Trigonometry topics to review:

- Basic identities
- Double –angle identities

## Calculus topics to review:

- Basic integration rules
- Techniques of integration: integrations by parts and partial fraction decomposition
- Improper integrals with infinite limits of integration
- Power series

## Sample Review Questions:

1. Solve these polynomial equations:

a)  $x^3 - x^2 - 4 = 0$

b)  $16x^4 + 24x^2 + 9 = 0$

c)  $x^5 + 5x^4 - 2x^3 - 10x^2 + x + 5 = 0$

2. Find the integral:

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$$

3. Use the properties of logarithms to solve for y:

$$\ln|y| = \ln|1 + x| + C \text{ where } C \text{ is an arbitrary constant.}$$

4. In each case, find A, B, and C so that the equation will be true:

$$\text{a) } \frac{1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

$$\text{b) } \frac{1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$$

$$\text{c) } \frac{1}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$$

5. Write the partial fraction decompositions for:

$$\text{a) } \frac{x-1}{x^2-x-2}$$

$$\text{b) } \frac{x}{(x-1)(x^2+x+1)}$$

$$\text{c) } \frac{-2x^3+2x^2-2x+6}{(x-1)^2(x^2-x+2)}$$

$$\text{d) } \frac{5x^3-3x^2+7x-3}{(x^2+1)^2}$$

6. Use appropriate trigonometric identities to find:

$$\text{a) } \int \sin 2t \cos 2t \, dt$$

$$\text{b) } \int \cos^2 t \, dt$$

7. Find the integral.

$$\text{a) } \int \frac{dx}{(x-3)(x+4)}$$

$$\text{b) } \int \frac{x-4}{(x-2)(x^2+1)} \, dx$$

$$c) \int \frac{5x^3 - 3x^2 + 7x - 3}{(x^2 + 1)^2} dx$$

$$d) \int e^x \sin x dx$$

$$e) \int x^2 e^{2x} dx$$

8. Determine whether each of the following integrals converges or diverges:

$$a) \int_0^{\infty} t e^{-t} dt$$

$$b) \int_0^{\infty} \frac{dt}{t^2 + 1}$$

9. Find the radius of convergence and interval of convergence for the given power series:

$$\sum_{i=1}^{\infty} \frac{(-1)^n}{10^n} (x - 5)^n$$

10. The function  $f(x) = \sin(x)\cos(x)$  is analytic at  $x = 0$ .

Find the first four terms of the corresponding power series in  $x$ .

**Sample Review Answers:**

$$1a) x = 2, \frac{-1 \pm i\sqrt{7}}{2}$$

$$1b) x_1 = \frac{i\sqrt{3}}{2}, x_2 = \frac{-i\sqrt{3}}{2}, x_3 = \frac{i\sqrt{3}}{2}, x_4 = \frac{-i\sqrt{3}}{2}$$

$$1c) x_1 = -1, x_2 = 1, x_3 = -1, x_4 = 1, x_5 = -5$$

$$2) \frac{x^2}{2} + x - \frac{2}{x-1} + \ln \left| \frac{x-1}{x+1} \right| + C$$

$$3) y = \pm e^C (1 + x)$$

$$4a) A = \frac{1}{2}, B = -1, C = \frac{1}{2}$$

$$4b) A = -1, B = 1, C = 1$$

$$4c) A = \frac{-1}{2}, B = \frac{-1}{2}, C = \frac{1}{2}$$

$$5a) \quad \frac{1/3}{x-2} + \frac{2/3}{x+1}$$

$$5b) \quad \frac{1}{3(x-1)} + \frac{-x+1}{3(x^2+x+1)}$$

$$5c) \quad \frac{2}{(x-1)^2} - \frac{3}{x-1} + \frac{x-4}{x^2-x+2}$$

$$5d) \quad \frac{5x-3}{x^2+1} + \frac{2x}{(x^2+1)^2}$$

$$6a) \quad \frac{-1}{8} \cos 4t + C$$

$$6b) \quad \frac{1}{2}t + \frac{1}{4}\sin 2t + C$$

$$7a) \quad \frac{1}{7} \ln \left| \frac{x-3}{x+4} \right| + C$$

$$7b) \quad \frac{1}{5} \left[ 9 \arctan x + \ln \left( \frac{x^2+1}{(x-2)^2} \right) \right] + C$$

$$7c) \quad \frac{5}{2} \ln(x^2 + 1) - 3 \tan^{-1} x - \frac{1}{x^2+1} + C$$

$$7d) \quad \frac{e^x}{2} (\sin x - \cos x) + C$$

$$7e) \quad \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C$$

8a) converges

8b) converges

$$9) \quad 10, (-5, 15)$$

$$10) \quad x - \frac{2}{3}x^3 + \frac{2}{15}x^5 - \frac{4}{315}x^7 + \dots$$

## **Resources available:**

### ***Help***

If you need any help, drop by or call the Mathematics Department (BHL 115, 992-2200) and someone there can guide you. In particular, you can get help from the following members of the mathematics department: Marina Frost, John Mitchell, Wes Orser, and Dennis Watson.

### ***Books to review***

- *College Algebra*, (5<sup>th</sup> ed., Thomson/Brooks-Cole), by Stewart, Redlin & Watson,

ISBN#: 0-495-75587-7

- *Trigonometry* (5<sup>th</sup> ed., Thomson/Brooks-Cole), by McKeague. ISBN#: 0-534-40392-1

- *Calculus Early Transcendental Functions* (4<sup>th</sup> ed., Houghton-Mifflin), by Larson/Hostetler/Edwards. ISBN #: 13:978-0-618-60624-5